# Solutions – Exam 1 Fall 2013

1.

(a) First, $D$ dominates $A$. Then $G$ dominates $E$. Then $C$ dominates $B$. Finally, $G$ dominates $H$ and no more eliminations are possible.

(b) There are two pure-strategy NE, $(C, F)$ and $(D, G)$. For finding the mixed equilibria, assume that player 1 plays $C$ with probability $p$ and $D$ with probability $1-p$, and that player 2 plays $F$ with probability $q$ and $G$ with probability $1-q$. In equilibrium, both players need to be indifferent between the two strategies, so that we get

$$\begin{aligned} 3q + 2 - 2q &= 2q + 4 - 4q \\ 3q &= 2 \\ q^* &= \frac{2}{3} \end{aligned}$$

so that player 1 is indifferent and

$$\begin{aligned} 3p &= p + 1 - p \\ 3p &= 1 \\ p^* &= \frac{1}{3} \end{aligned}$$

so that player 2 is indifferent. The mixed-strategy NE is hence $\left(\left(\frac{1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{1}{3}\right)\right)$ or $(p^*, q^*) = \left(\frac{1}{3}, \frac{2}{3}\right)$.

(c) If $a = 2$, the game $G$ has the unique Nash Equilibrium $(C, F)$. This will be played in both rounds, so that the unique SPNE of $G$ is

$$(CCCCCCCCCCCCCCCCC, FFFFFFFFFFFFFFFFF)$$

(that is 17 times $C$ and $F$ each – one choice for the first round and one choice each after every possible outcome of the first round).

(d) The trigger strategies are:

- For player 1: Play $A$ in the first round and if the outcome of all previous rounds has been $(A, E)$, otherwise play $C$.
- For player 2: Play $E$ in the first round and if the outcome of all previous rounds has been $(A, E)$, otherwise play $F$.

These strategies constitute a SPNE if it is not profitable for any of the players to deviate, i.e. if continued cooperation is better than the one-shot deviation followed by playing the stage-game NE forever. Note that the optimal one-shot deviations of both players, $D$ (for player 1) and $H$ (for player 2) both give a payoff of 9. If the players play the trigger strategies, they both get a payoff of 8 every round. If they play the stage-game NE, they get a payoff of 3 every round.

The trigger strategies therefore constitute an equilibrium if

$$\begin{aligned} 8(1+\delta+\delta^2+\dots) &\geq 9+3(\delta+\delta^2+\dots) \\ \frac{8}{1-\delta} &\geq 9+\frac{3\delta}{1-\delta} \\ \delta \geq \delta^* &= \frac{1}{6}. \end{aligned}$$

2. The correct ordering is:

   - Iterated Elimination of Strictly Dominated Strategies
   - Nash Equilibrium
   - Subgame-Perfect Nash Equilibrium
   - Perfect Bayesian Nash Equilibrium
   - Perfect Bayesian Equilibrium with signaling requirements 5 and 6

3.

   (a) Since $R$ strictly dominates $L$ for $t_1$, there can be no PBE in which $t_1$ plays $L$. Therefore, the only possible separating PBE is the one where the sender plays $(R, L)$, and we find the PBE $((R, L), (u, u), p = 0, q = 1)$.

   (b) The pooling PBE is $\left((R, R), (d, u), p \geq \frac{1}{3}, q = \frac{1}{2}\right)$. Signaling requirement 5 states that the receiver does not believe that strictly dominated messages are played with positive probability. Here, this translates to the requirement that $p = 0$, so that there is no pooling PBE in this game that fulfills signaling requirement 5.

   (c) The answer should show that the student has understood the main ideas of signaling theory. In part (i), the answer should give a signal that can easily be imitated by someone who is not strong, such as simply saying "I am strong." A credible signal is a signal that a weak person can not imitate (or only at very hight cost), such as lifting a heavy weight, beating someone in an arm-wrestling contest or other physical exercises. Since these cannot be imitated by the weak person, the other person (the receiver) knows that only a strong person could have sent the signal, and therefore believes that the sender is a strong person.

4.

(a) The profit of firm 1 is

$$\pi_1(q_1, q_2) = (15 - q_1 - q_2 - 3)q_1.$$

The first-order condition (FOC) gives a best-response function of

$$q_1 = \frac{12 - q_2}{2}$$

and since the two firms are symmetric, we get equilibrium quantities and profits of

$$\begin{aligned} q_1^{NE} = q_2^{NE} &= 4 \\ \pi_1^{NE} = \pi_2^{NE} &= 16. \end{aligned}$$

(b) Firm 2's optimal reaction function is the same as in (a):

$$q_2 = \frac{12 - q_1}{2}.$$

Firm 1 takes this as given, so that the profit of firm 1 is

$$\pi_1(q_1) = \left(15 - q_1 - \frac{12 - q_1}{2} - 3\right) q_1$$

which gives the FOC

$$15 - 2q_1 - \frac{12 - 2q_1}{2} - 3 = 0$$

or $q_1^{seq} = 6$. The equilibrium quantity for firm 2 is then $q_2^{sec} = 3$ (note that this is not the equilibrium *strategy* of firm 1 – that is given by the reaction function above). Equilibrium profits are $\pi_1^{seq} = 18$ and $\pi_2^{seq} = 9$.

(c) Yes, firm 1 is better off than in the static game (and better off than firm 2). Since firm 1 (the leader) knows that firm 2 will learn $q_1$ before making its own choice, it can take the reaction of firm 2 as given and choose a higher quantity than in equilibrium. This forces firm 2 to choose a lower quantity. [Another point, which is however not essential to answering the question: From the fact that firm 1 could have chosen $q_1^{NE}$ also in the sequential game, but didn't, we can immediately see that firm 1 must be better off than in the static game and that there is a first-mover advantage.]

(d) There is a first-mover disadvantage in all zero-sum games and some other games where a dynamic game allows the second player to profitably "react" to the first player's choice. There can never be a first-mover disadvantage if the static game has a NE in pure strategies, because then the first player can just play his equilibrium strategy and the second player will optimally react with his equilibrium strategy. The players then simply play the NE of the static game.